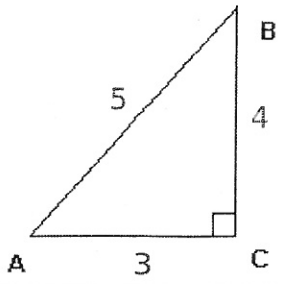
Advanced Algebra Trig Reminders for the ACT

Right triangle trig:

Sine = $\frac{opposite}{hypotenuse}$ Cosine = $\frac{adjacent}{hypotenuse}$ Tangent = $\frac{opposite}{adjacent}$

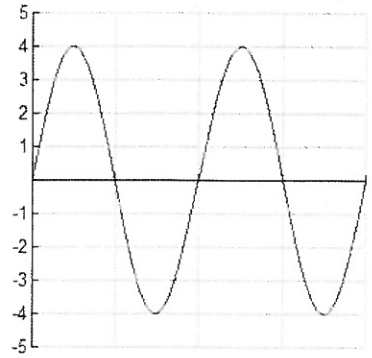
For example:



sinA = $\frac{4}{5}$ cosA = $\frac{3}{5}$ tanA =

On the coordinate plane:

Sine = $\frac{y}{r}$ Cosine = $\frac{x}{r}$ Tangent = $\frac{y}{x}$



A **sinusoid** is a cyclic graph, where the input is the angle measure and the output is the trig ratio (for either sine or cosine).

Amplitude: The distance from the maximum to midline (or the distance from the minimum to midline)

Period: How long it takes to get through one cycle.

**Note: 360 degrees is equal to 2π radians

Solving formulas in terms of another variable.

For example: Solve a(b + c) = d for b.

Distribute: ab + ac = d

Subtract ac from both sides of the equation: ab = d – ac

Divide by a: $b = \frac{d-ac}{a}$ which can also be written as $b = \frac{d}{a} - c$

A **matrix** (plural matrices) is a rectangular array of numbers displayed in rows and columns.

For example:

$\begin{bmatrix} 2 & 7 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$ is a matrix with dimensions 3×2, because there are three rows and 2 columns.

Function notation

For example: Given f(x) = 2x +1, find f(3).

Then 2(3) + 1, so it equals 7.

Know how to evaluate expressions involving exponents, roots, absolute value

Solve and graph linear equations, linear inequalities, and systems of linear equations.

Solve quadratic equations using the quadratic formula or factoring

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

A **logarithm** is an exponent.

For example: $\log_2 8$ is equal to 3, because $2^3 = 8$.

Sets of numbers: complex/imaginary, real, irrational, rational, integers, whole

Extended Distributive property/Factoring

Multiply a binomial times a binomial. For example: Multiply (x + 3)(x + 5). Answer: $x^2 + 8x + 15$

Square a binomial. For example: Multiply $(x + y)^2$. Answer: $x^2 + 2xy + y^2$

Factoring is the inverse of the distributive property. For example: Factor $x^2 + 8x + 15$. Answer: (x + 3)(x + 5).

15. What is the sum of the 2 solutions of the equation: $x^2 + 2x - 15 = 0$?

A. -15
B. -5
C. -2
D. 0
E. 3

$(x+5)(x-3)=0$

$x+5=0$ or $x-3=0$

$x=-5$ $\quad x=3$

$-5+3=-2$

16. For a certain quadratic equation, $ax^2 + bx + c = 0$, the 2 solutions are $x = \frac{3}{5}$ and $x = -\frac{1}{2}$. Which of the following could be factors of $ax^2 + bx + c$?

A. (5x – 3) and (2x + 1)
B. (5x – 1) and (2x +3)
C. (5x +1) and (2x – 3)
D. (5x + 3) and (2x – 1)
E. (5x + 3) and (2x + 1)

$5x-3=0$
$5x=3$
$x=3/5$

Check by setting factors $=0$

17. Given $f(x) = 4x + 1$ and $g(x) = x^2 - 2$, which of the following is an expression for $f(g(x))$?

A. $-x^2 + 4x + 1$
B. $x^2 + 4x - 1$
C. $4x^2 - 7$
D. $4x^2 - 1$
E. $16x^2 + 8x - 1$

$f(x^2-2)=4(x^2-2)+1$

$=4x^2-8+1$

18. If $f(x) = x^2 - 2$, then $f(x + h) = ?$

A. $x^2 + h^2$
B. $x^2 - 2 + h$
C. $x^2 + h^2 - 2$
D. $x^2 + 2xh + h^2$
E. $x^2 + 2xh + h^2 - 2$

$f(x+h)=(x+h)^2-2$

$x^2+2xh+h^2-2$

19. In the complex numbers, where $i^2 = -1$, $\frac{i}{1+i} \cdot \frac{1-i}{1-i} = ?$

A. $i - 1$
B. $1 + i$
C. $1 - i$
D. $\frac{1-i}{2}$
E. $\frac{1+i}{2}$

$\frac{i-i^2}{1-i^2} = \frac{i-(-1)}{1-(-1)}$

$= \frac{i+1}{2}$

20. What value of n will satisfy the equation $0.1(n + 1{,}350) = n$?

A. 1,500
B. 1,485
C. 1,215
D. 150
E. 135

$.1n+135=n$

$135=.9n$

21.) Which of the following gives all the solutions of $x^2 + 2x = 8$?

A. 4 and -2
B. -4 and 2
C. -8 and 1
D. -4 only
E. -8 only

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$-4, 2$

22. Which of the following is an irrational number that is a solution to the equation $|x^2 - 12| - 4 = 0$?

A. 4
B. $\sqrt{2}$
C. $2\sqrt{2}$
D. $4\sqrt{2}$
E. $2\sqrt{3}$

$|x^2-12|=4$

$x^2-12=4$ or $x^2-12=-4$

$x^2=16$ $\quad x^2=8$

$x=4,-4$ (NO) $\quad x=\pm\sqrt{8}$

23. What is the real value of x in the equation $\log_2 24 - \log_2 3 = \log_5 x$?

A. 3
B. 21
C. 72
D. 125
E. 243

$\log_2\left(\frac{24}{3}\right)=\log_5 x$

$\log_2 8=\log_5 x$

$3=\log_5 x \rightarrow 5^3=x$

24. The value of $\log_5\left(5^{\frac{13}{2}}\right)$ is between of the following pairs of consecutive integers?

A. 0 and 1
B. 4 and 5
C. 5 and 6
D. 6 and 7
E. 9 and 10

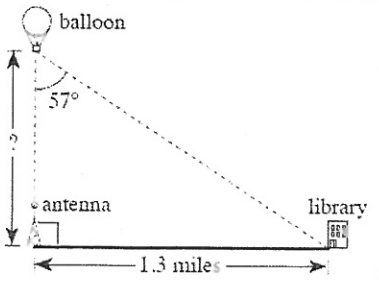
$5^{\square}=5^{\frac{13}{2}}$

25. What is the real value of x in the equation: $\log_2 24 - \log_2 3 = \log_5 x$?

A. 3 B. 21 C. 72 D. 125 E. 243

same as # 23 !!

26. From a hot air balloon, the angle between a radio antenna straight below and the base of the library downtown is 57° as shown below. If the distance between the radio antenna and the library is 1.3 miles, how many miles high is the balloon?



A. $\frac{1.3}{\sin 57°}$

B. $\frac{1.3}{\cos 57°}$

C. $\frac{1.3}{\tan 57°}$

D. $1.3 \sin 57°$

E. $1.3 \tan 57°$

$\tan 57° = \frac{1.3}{x}$

$\tan 57° \cdot (x) = 1.3$

$x = \frac{1.3}{\tan 57°}$

27. A 24 foot ladder is leaning against a telephone pole. The angle of elevation to the top of the telephone pole is 37°. What is the height, in feet, of telephone pole?

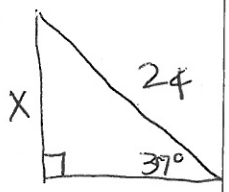
A. 24tan37°

B. 24sin37°

C. 24cos37°

D. 24sec37°

E. 24cot37°



$\sin 37° = \frac{x}{24}$

$x = 24(\sin 37°)$

28. For right triangle Δ RST shown below, what is tan R?

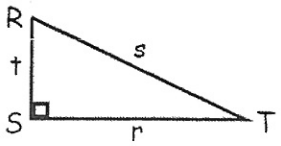
A. r/s

B. r/t

C. t/r

D. t/s

E. s/t



29. Which of the following is equivalent to $\frac{\tan n \csc n}{\sin n \sec n}$?

(A) 1

(B) $\sin n$

(C) $\cos n$

(D) ~~$\cot n$~~

(E) $\csc n$

$\frac{\sin n}{\cos n} \cdot \frac{1}{\sin n}$

$\sin n \cdot \frac{1}{\cos n}$

$\frac{\frac{1}{\cos n}}{\frac{\sin n}{\cos n}} = \frac{1}{\cos n} \cdot \frac{\cos n}{\sin n}$

$= \frac{1}{\sin n} = \csc n$

30. If $\tan A = \frac{x}{y}$, $x > 0$, $y > 0$, and $0 < A < 90°$, what is sinA?

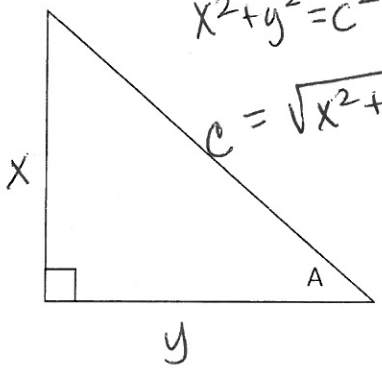
A. $\frac{x}{y}$

B. $\frac{y}{x}$

C. $\frac{x}{\sqrt{x^2+y^2}}$

D. $\frac{y}{\sqrt{x^2+y^2}}$

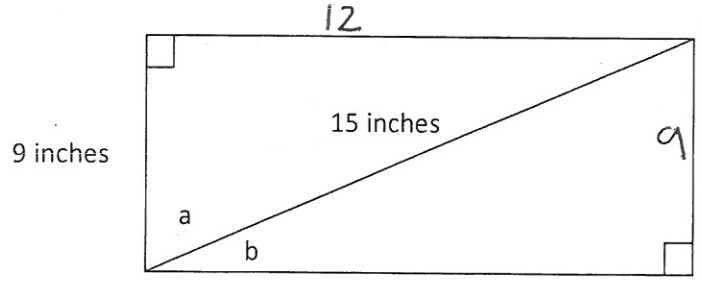
E. $\frac{\sqrt{x^2+y^2}}{x}$



$x^2+y^2=c^2$

$c = \sqrt{x^2+y^2}$

31. The 2 triangles in the rectangle below share a common side. What is sin(a –b)?
(Note: sin(a – b) = sinacosb – cosasinb for all a and b)



A. 7/25

B. 1/2

C. 3/5

D. 1

E. 25/9

$\frac{12}{15} \cdot \frac{12}{15} - \frac{9}{15} \cdot \frac{9}{15}$

$\frac{4}{5} \cdot \frac{4}{5} - \frac{3}{5} \cdot \frac{3}{5}$

$\frac{16}{25} - \frac{9}{25}$

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Hmmmm.....

32. The quantity $\sqrt[n]{2^p}$ is defined when n is an integer greater than 2 and p is any nonzero real number. Which of the following is a relationship between n and p that will always make $\sqrt[n]{2^p}$ a positive integer? $2^{\frac{p}{n}}$ pos int? A. $\frac{p}{n}$ is a positive integer B. $\frac{n}{p}$ is a positive integer C. p is greater than n D. n is greater than p E. The sum of p and n is one	33. Which of the following statements *must* be true whenever n, a, b, and c are positive integers such that $n < a$, $c > a$, and $b > c$? $n < a < c < b$ A. $a < n$ B. $b - n > a - n$ C. $b < n$ D. $n + b = a + c$ E. $2n > a + b$
34. Which of the following is the set of all real numbers x such that $x + 3 > x + 5$? $3 > 5$ A. The empty set B. The set containing all real numbers C. The set containing all negative real numbers D. The set containing all nonnegative real numbers E. The set containing only zero	35. How many irrational numbers are there between 1 and 6 ? **F.** 1 **G.** 3 **H.** 4 **J.** 10 **K.** Infinitely many
36. If $6a^4b^3 < 0$, then which of the following CANNOT be true? a^4 will always be > 0 so b^3 must be < 0 $b < 0$ A. $b < 0$ B. $b > 0$ C. $a = b$ D. $a < 0$ E. $a > 0$	37. Each of the variables t, w, x, y, and z represents a different *positive* real number. Given the equations below, which of the 4 variables w, x, y, and z necessarily has the greatest value? $1.23w = t$ $1.01x = t$ $0.99y = t$ $0.23z = t$ **F.** w **G.** x **H.** y **J.** z **K.** Cannot be determined from the given information

And one more matrix problem!

In the 2 × 2 matrix below, b_1 and b_2 are the costs per pound of bok choy (Chinese greens) at Market 1 and Market 2, respectively; r_1 and r_2 are the costs per pound of rice flour at these 2 markets, respectively. In the following matrix product, what does q represent?

$$[0.5 \quad 0.5] \cdot \begin{bmatrix} b_1 & r_1 \\ b_2 & r_2 \end{bmatrix} = [p \quad q]$$

F. The cost of r_1 pounds of rice flour at \$0.50 per pound
G. The cost of a half-pound of rice flour at Market 1
H. The total cost of a half-pound of bok choy and a half-pound of rice flour at Market 1
J. The total cost of a half-pound of bok choy and a half-pound of rice flour at Market 2
K. The total cost of a half-pound of rice flour at Market 1 and a half-pound of rice flour at Market 2

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